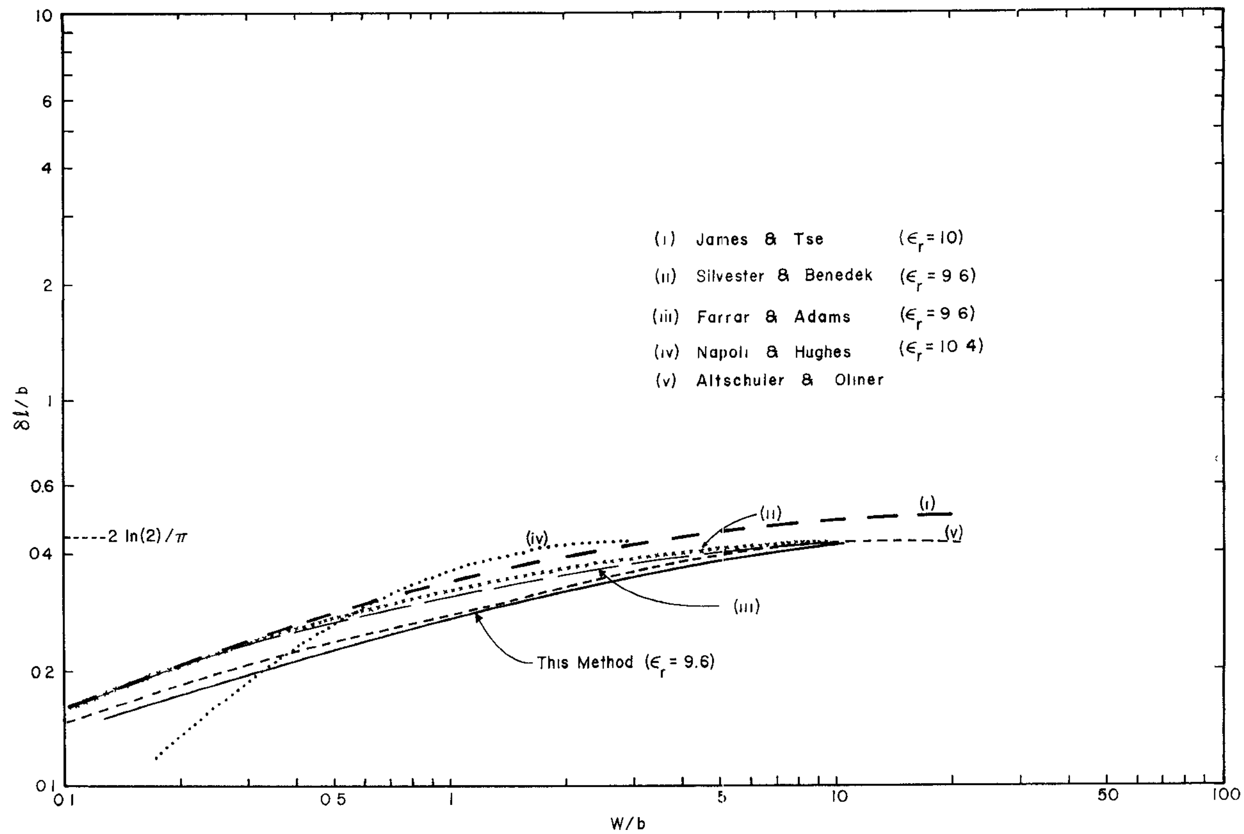

Fig. 2. End effect of a semi-infinite microstrip line.

[6], [7]. This quantity was calculated by using the expressions for C_{ex} given earlier, and the results obtained are shown in Fig. 2. The same quantity has been computed by James and Tse [7] by using an alternative approach, and their results are also included in Fig. 2 for ease of comparison. It is evident that the numerical results obtained in this short paper compare favorably with Farrar and Adams, as well as others.

It may be useful to quote some typical computation time for calculating $C(l)$ by (12). Typical time of the CDC G-20 computer was about 60 s for this calculation (execution time). The above computer is approximately ten times slower than the IBM 360/75. To minimize the computation time for C_{ex} given in (14), the choice of l is important. A numerical experiment shows that, if $l \gtrsim 10$ W, $C(l)$ increases linearly with l. Hence the limiting process can be omitted for this choice of l. Furthermore, since the computation of C_0 requires less than 5 s (execution time), the computation time of C_{ex} is also about 60 s.

In conclusion, the method described in this short paper has many advantages, one of which is its numerical efficiency. Another feature is that it is quite general, since many other types of junctions and finite structures can be solved by the present method, either in its present form or with some modifications. Some examples of such structures are gaps in the uniform strip, T junction, etc., that are currently under investigation.

Finally, it should be mentioned that Maeda [8] has recently reported a method for analyzing the gap structure in the microstrip line. The approach outlined in this short paper is believed to be numerically more efficient, since the expression for Green's function in the transform domain is a closed form in contrast to a slowly converging series in the space domain.

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A Quasi-Dynamic Method of Solution of a Class of Waveguide Discontinuity Problems

LEONARD LEWIN AND JAMES P. MONTGOMERY

***Abstract*—It is shown that, if expansion terms of *all* the modes appearing in the Green's function for the problem are retained, the singular integral equation method can be made to apply by generating a differential equation for this integral. The solution of the differential equation is straightforward, and the inversion of the resulting integral equation then follows standard methods. The process is applied in detail to the case of the capacitive diaphragm, and the results compared to the quasi-static method with correction terms. The results are close for small guide widths, but the present method should give superior results if the guide width permits some overmoding.**

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Introduction

The class of problems referred to is that in which the solution can be formulated in terms of a transversely varying aperture field or obstacle current. Typically, it includes inductive and capacitive diaphragms, gratings, step discontinuities, bifurcations, inhomogeneous waveguide junctions, and transversely magnetized ferrite boundaries.

Early researchers used integral equation formulations [1], [2], sometimes coupled with solutions from known electrostatic problems [1]-[3] to establish a solution. These solutions are known as *quasi-static* solutions and they depend essentially on approximations of the type $\Gamma_n=(n^2\pi^2/b^2-k^2)^{1/2}\sim n\pi/b$ for the attenuation constant for higher order modes. The approximation is poorest for the lowest order modes, and correction terms involving the small quantity $(\Gamma_n-n\pi/b)$ for small n can also be incorporated into the solution.

An extension of the method using singular integral equations [4] still was based on treating the lowest order modes preferentially in this way. At the other extreme [5], quasi-optical methods based on the high-frequency limit calculate the diffraction from edge discontinuities. Mittra [6] using a modified residue calculus technique, has extended the Wiener–Hopf type of solution to obtain good approximations to some waveguide configurations.

The present short paper indicates a somewhat different approach based on an approximate form for the attenuation constants to be used for *all* the higher order modes. The relation $(b\Gamma_n/n\pi)=1-(kb/n\pi)^2/2-(kb/n\pi)^4/8\cdots$ enables the kernel of the integral equation to be expressed as a truncated series in powers of $(kb/\pi)^2$ to which all the higher modes contribute. For want of a better name this method will be called *quasi-dynamic*. As a further refinement it is still possible to incorporate higher order correction terms involving the small quantity $(b\Gamma_n/n\pi-1-(kb/n\pi)^2/2-\cdots)$ for small n; and this must in any case be done if kb is large enough to permit a lower order mode to propagate. The expansion for large n is still valid and useful, even if kb is not all that small.

The Form of the Kernel

The difficulty in extending the singular integral equation method to the more complex kernels here considered arises from the fact that sums of the form $\sum_{n=1}^{\infty}\cos n\theta/n^{2r}$ are transcendental functions (Clausen functions) for $r>0$, with r integral, and it is quite impractical to make needed transformations like $\cos\theta=c+s\xi$ on a term-by-term basis. The clue to a method of dealing with them springs from the observation that application of the operator $(d/d\theta)^{2r}$ reduces them to the $r=0$ form, which is tractable. Hence if the series for Γ_n is truncated after N terms, the problem can be reduced essentially to one of solving a differential equation of order $2N$ with constant coefficients. As with the quasi-static methods, to which the present method obviously has an affinity, the real problem then becomes one of evaluating the various constants that arise, and which here appear in the form of integrals involving the Clausen functions. Whereas, in the quasi-static method, the integrals give rise to elliptic functions, with trigonometric and logarithmic forms as special cases, in the present instance the functions involved are the polylogarithms [7], which, fortunately, are easy to handle and are well tabulated.

As an example to illustrate this method we solve the problem of a symmetrical capacitive diaphragm, and compare the result to the quasi-static solution with correction terms. Although the intermediate stages seem somewhat formidable, it is encouraging to find that the final expression for the capacitance is a quite simple expression, which can be readily related to the quasi-static solution. The form of aperture field is also relatively simple.

The results of this method are likely to be most useful for calculating aperture fields, mode coupling coefficients, and obstacle parameters in situations in which the quantity kb lies in the small to intermediate range, i.e., low-frequency to slightly overmoded conditions. Regular gratings are similarly analyzed, using Floquet's theorem to express the fields in terms of a single grating characteristic.

Capacitive Diaphragm Solution

Fig. 1 shows a parallel plate arrangement of spacing b with a diaphragm with aperture from $y=d$ to $b-d$.

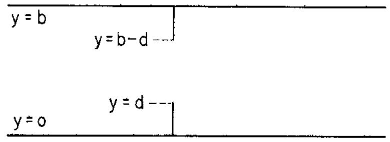

Fig. 1. Capacitive diaphragm.

The transition to rectangular guide of width a can be made by replacing k by $k'=k(1-\lambda^2/4a^2)^{1/2}$ at the end of the analysis. Similarly, Γ_n' and α' have λ replaced by λ_g in their definitions. Writing $\beta=\pi d/b$, $\theta=\pi y/b$, $\phi=\pi y'/b$, and $\Gamma_n=(n^2\pi^2/b^2-k^2)^{1/2}$ the equation to be solved for the aperture field $E(\phi)$ can be put in the form [4]

$$1=\frac{1}{\pi}\int_\beta^{\pi-\beta}E(\phi)d\phi+\frac{j2k}{\pi}\int_\beta^{\pi-\beta}E(\phi)\sum_1^\infty\frac{\cos n\theta\cos n\phi}{\Gamma_n}d\phi,\qquad \beta<\theta<\pi-\beta. \tag{1}$$

The reflection coefficient R in the guide is given by

$$1+R=\frac{1}{\pi}\int_\beta^{\pi-\beta}E(\phi)d\phi \tag{2}$$

and the normalized susceptance B, one of the objects of our calculation, is related to R by

$$B=2jR/(1+R). \tag{3}$$

To solve (1), differentiate with respect to θ, and replace $n\pi/b\Gamma_n$ by the second-order approximation

$$n\pi/b\Gamma_n\approx 1+\alpha^2/n^2,\qquad \text{where } \alpha^2=2b^2/\lambda^2. \tag{4}$$

Then (1) becomes

$$\int_\beta^{\pi-\beta}E(\phi)\sum_1^\infty\sin n\theta\cos n\phi(1+\alpha^2/n^2)d\phi=0,\qquad \beta<\theta<\pi-\beta. \tag{5}$$

In the absence of the term in α^2/n^2 this would be of the same form as appears in the singular integral equation method. In the present instance, however, the corresponding expression will not be zero, but some function of θ, $g(\theta)$ say, where we expect $g(\theta)$ to be of order α^2. Hence we can write

$$\int_\beta^{\pi-\beta}E(\phi)\sum_1^\infty\sin n\theta\cos n\phi d\phi=g(\phi),\qquad \beta<\theta<\pi-\beta. \tag{6}$$

If we knew $g(\theta)$, (6) could be solved by standard techniques. To find $g(\theta)$, subtract (5) from (6).

$$-\alpha^2\int_\beta^{\pi-\beta}E(\phi)\sum_1^\infty\frac{\sin n\theta\cos n\phi}{n^2}d\phi=g(\theta). \tag{7}$$

Differentiating twice, and using (6) gives

$$\alpha^2 g=d^2g/d\theta^2. \tag{8}$$

Hence

$$g(\theta)=A\sinh[\alpha(\theta-\pi/2)]+A'\cosh[\alpha(\theta-\pi/2)] \tag{9}$$

with A and A' to be determined. The solution is put in this form because, in the present case of the symmetrical diaphragm, (1) is symmetrical about the guide center $\theta=\pi/2$, and hence $g(\theta)$, which is related to (1) by a differentiation, is antisymmetrical. By inspection A' is zero, and, consistent with our expansion in powers of α, only the first power of an expansion of (9) in α need be retained. Hence

$$g(\theta)=A\alpha(\theta-\pi/2) \tag{10}$$

to $O(\alpha^4)$ since, as will be shown, $A=O(\alpha)$.

Equation (6) is now ready for solution by the singular integral equation technique, using the relation

$$\sum_1^\infty\sin n\theta\cos n\phi=\frac{1}{2}\,\frac{\sin\theta}{\cos\phi-\cos\theta}. \tag{11}$$

(As discussed in [4] the use of the above equation requires that certain integrals arising should be taken as principal values. This is implicit in what follows.)

Substituting (11) into (6) and writing

$$\cos\phi=s\eta,\quad \cos\theta=s\xi,\quad s=\cos(\pi d/b)$$

and

$$E(\phi)d\phi=F(\eta)d\eta \tag{12}$$

we get

$$\int_{-1}^{1} \frac{F(\eta)d\eta}{\eta - \xi} = \frac{-2sg(\theta)}{\sin\theta}, \qquad -1 < \xi < 1. \tag{13}$$

The solution is

$$F(\eta) = \frac{1}{(1-\eta^2)^{1/2}}\left[C' + \frac{2\alpha s A}{\pi^2}\int_{-1}^{1}\frac{(\theta - \pi/2)}{\sin\theta}\,\frac{(1-\xi^2)^{1/2}}{\xi - \eta}d\xi\right] \tag{14}$$

with C' arbitrary and θ given by (12).

The integration is performed in the Appendix, and the constant A is there derived by taking the limiting value from (7) as $\theta \rightarrow \pi/2$. In terms of $E(\theta)$, and a new constant C, (14) becomes

$$E(\phi) = C\left[\frac{\sin\phi}{(s^2 - \cos^2\phi)^{1/2}} + \alpha^2 \log s \log\frac{\sin\phi + (s^2 - \cos^2\phi)^{1/2}}{(1-s^2)^{1/2}}\right]. \tag{15}$$

Apart from a determination of C, (15) gives the form of the aperture field as a function of $\phi = \pi y/b$.

To determine C we must return to (1), the undifferentiated form of the integral equation, insert (15) for $E(\phi)$, and carry out the indicated integrations. It is here that the calculations, outlined in the Appendix, become a little testing. With C determined (2) gives R and (3) the susceptance. The final result, after replacing k by k' (or λ by λ_g) is

$$B = \frac{4b}{\lambda_g}\left(-\log s + \frac{b^2}{2\lambda_g^2}K\right) \tag{16}$$

where

$$K = Li_3(1) - Li_3(s^2) + 2\log s Li_2(s^2). \tag{17}$$

The polylogarithms [7], which are well tabulated, are defined by

$$Li_2(x) = \sum_1^\infty x^n/n^2 = -\int_0^x \frac{\log(1-x)}{x}dx$$

$$Li_3(x) = \sum_1^\infty x^n/n^3 = \int_0^x \frac{Li_2(x)}{x}dx. \tag{18}$$

Comparison with the Quasi-Static Results

The relevant formula with two correction terms included can be found in [2, eq. (3.119)]. Taking the δ_n correction coefficients small so that their products can be neglected, the expression to compare with (17) has K replaced by K_0 where

$$K_0 = (1-s^2)^2[1 + (3s^2 - 1)^2/8]. \tag{19}$$

Equation (17) is plotted in Fig. 2 and, on an enlarged scale, so is the difference $K - K_0$. It is seen that numerically the two expressions behave quite similarly. The difference is greatest when s is small, since more higher order modes are then generated.

Higher Order Solutions

Equation (16) can be improved in two directions. More terms could be retained in the expansion (4). Alternatively, or in addition, the right-hand side of (5) could be augmented with the exact form of the lowest order modes less the dominant expansion terms already considered in (4). For example, in the case of the symmetric capacitive diaphragm, the right-hand side of (5), instead of zero, could be (since odd-n terms integrate to zero),

$$-\Delta_2\int_\beta^{\pi-\beta} E(\phi)\sin 2\theta \cos 2\phi d\phi$$

with

$$\Delta_2 = (2\pi/b\Gamma_2' - 1 - \alpha'^2/4). \tag{20}$$

(The transition to k' and λ_g has been anticipated.) This term is $O(\alpha^4)$ and it would not be consistent to retain products of higher order. Hence $E(\phi)$ in (20) need be only the first term in (15), unless b is large enough for the $n=2$ mode to propagate, when the additional $O(\alpha^2)$ term does need to be retained. Apart from this case, the extra integrations needed to cope with (20) have already been done in obtaining [2, eq. (3.119)], and quoting from there, with the sole difference of using Δ_2 instead of δ_2 we get an addition to the expression in the brackets in (16) of

$$\Delta_2(1-s^2)^2/(1+\Delta_2 s^4). \tag{21}$$

The use of more terms in (4) would lead to expressions like $n\pi/b\Gamma_n$ $\approx 1 + \alpha^2/n^2 + 3\alpha^4/8n^4$, for example. The equation for $g(\theta)$ would become

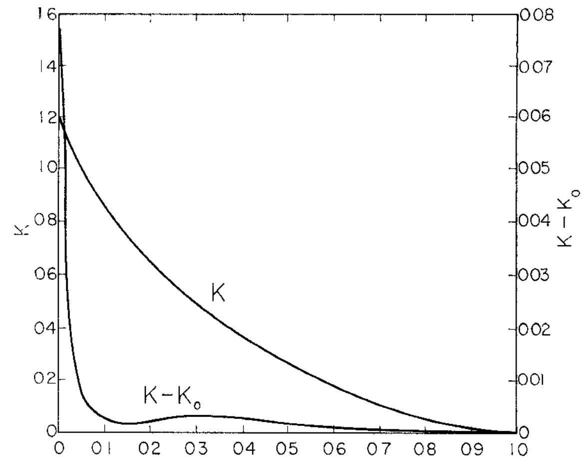

Fig. 2. Coefficients K and K_0 as a function of s.

$$d^4g/d\theta^4 - \alpha^2 d^2g/d\theta^2 + 3\alpha^4 g/8 = 0. \tag{22}$$

To order α^6 this gives

$$g(\theta) = A\alpha(\theta - \pi/2) + B\alpha^3(\theta - \pi/2)^3. \tag{23}$$

The solution then proceeds as before, except that there are more integrals to be evaluated, and more constants to be determined. The constant A will be of order α and will contain an α^3 term. B will be of order α. Clearly $g(\theta)$ is a polynomial in θ and it is unnecessary to actually obtain the differential equation in order to find it, as it can be written down by inspection. But as with the quasi-static method, if too many terms are retained, the large number of constants to be evaluated makes the process rather cumbersome.

The Inductive Case

The method applies, with some changes, to inductive configurations. The quantity α^2 in (4) becomes replaced by $-\gamma^2$, with $\gamma^2 = 2a^2/\lambda^2$, and (9) is replaced by

$$g(\theta) = A\sin[\gamma(\theta - \pi/2)] + A'\cos[\gamma(\theta - \pi/2)]. \tag{24}$$

Since for a typical guide operated in the dominant mode, $2a^2/\lambda^2$ is near unity, it is better to expand around this point. Thus for the symmetrical case, for which the A' term in (24) is required, the approximation to $g(\theta)$ would give

$$g(\theta) \approx A'\sin\theta + A'(\theta - \pi/2)(2a^2/\lambda^2 - 1). \tag{25}$$

The analysis then proceeds much as before.

Comparison with Other Techniques

Shestopalov [8] has analyzed waveguide diaphragms by a method which, although apparently differing from the singular integral equation approach, is really a variant of it. He retains N higher order modes and solves for them with Nth-order determinants whose elements are related to Fourier components of the aperture field. Hence in terms of the amount of work involved, the additional retention of higher order modes in the present method should be comparable.

It is perhaps worth pointing out that in the special case of waveguide diaphragms the properties of Legendre polynomials can be used to simplify and compact some of the formulas, and that many of Shestopalov's papers make good use of this. The real grind in problems of this sort is in expressing the integrals involved in terms of tabulated functions. This has always seemed worth doing where it was possible, but with modern computational tools available the necessity has receded somewhat.

Appendix

The integration of (14) involves the quantity $(\theta - \pi/2)$ cosec θ, with $\cos\theta = s\xi$. A tractable expression uses the subsidiary result, which is readily verified,

$$\frac{\theta - \pi/2}{\sin\theta} = -s\xi\int_0^{\pi/2}\frac{\cos\psi d\psi}{1 - s^2\xi^2\cos^2\psi} \tag{A1}$$

and the relevant integral becomes

$$-\int_{-1}^{1}\int_{0}^{\pi/2}\frac{(1-\xi^2)^{1/2}}{\xi-\eta}\,\frac{s\xi\cos\psi d\psi d\xi}{1-s^2\xi^2\cos^2\psi}. \quad \text{(A2)}$$

In the integrand add and subtract the expression $s\eta\cos\psi/(1-s^2\eta^2\cos^2\psi)$. The first term becomes

$$-\int_{0}^{\pi/2}\frac{s\eta\cos\psi d\psi}{1-s^2\eta^2\cos^2\psi}\int_{-1}^{1}\frac{(1-\xi^2)^{1/2}d\xi}{\xi-\eta}$$

which integrates to $s\pi\eta(\pi/2-\phi)$ cosec ϕ on using (A1) and known results. The second term simplifies to

$$-\int_{0}^{\pi/2}\frac{s\cos\psi}{1-s^2\eta^2\cos^2\psi}\left[\int_{-1}^{1}\frac{(1-\xi^2)^{1/2}d\xi}{1-s^2\xi^2\cos^2\psi}\right]d\psi = -\frac{\pi}{s}\int_{0}^{\pi/2}\frac{[1-(1-s^2\cos^2\psi)^{1/2}]}{\cos\psi(1-s^2\eta^2\cos^2\psi)}d\psi$$

on carrying out the ξ integration. This can now be integrated by taking $\sin\psi$ as a new variable and using standard techniques.

The evaluation of A in (14) requires the use of (7). Since both sides vanish at $\theta=\pi/2$, the limit can be taken by L'Hospital's rule, to give, after some simplification

$$A = \frac{\alpha C}{2}\int_{-1}^{1}\frac{d\eta}{(1-\eta^2)^{1/2}}\log(2s\eta) = (\alpha C\pi/2)\log s. \quad \text{(A3)}$$

On inserting (15) into (1) and keeping terms to $O(\alpha^4)$, five integrals appear. Some are relatively straightforward and some can be obtained by differentiation with respect to s, simplification, and then reintegration. The one requiring more attention is

$$F(s) = \int_{-1}^{1}\frac{d\eta}{(1-\eta^2)^{1/2}}\sum_{1}^{\infty}\frac{\cos n\phi\cos n\pi/2}{n^3} \quad \text{(A4)}$$

in which the value of θ has been chosen as $\pi/2$. An integration by parts reduces it to a consideration of

$$F_1(s) = \int_{0}^{\sin^{-1}s}\sin^{-1}\left(\frac{\sin\psi}{s}\right)\sum_{1}^{\infty}\frac{\sin 2\psi}{n^2}d\psi \quad \text{(A5)}$$

where $\phi=\psi+\pi/2$. Now $\sin^{-1}(u)$ for $u>1$ is $\pi/2$ plus an imaginary term. Hence (A5) can be written

$$F_1(s) = \operatorname{Re}\int_{0}^{\pi/2}\sin^{-1}\left(\frac{\sin\psi}{s}\right)\sum_{1}^{\infty}\frac{\sin 2\psi}{n^2}d\psi - \int_{\sin^{-1}s}^{\pi/2}\frac{\pi}{2}\sum_{1}^{\infty}\frac{\sin 2\psi}{n^2}d\psi. \quad \text{(A6)}$$

The second term cancels an integral arising elsewhere, and on putting

$$F_2(s) = \int_{0}^{\pi/2}\sin^{-1}\left(\frac{\sin\psi}{s}\right)\sum_{1}^{\infty}\frac{\sin 2\psi}{n^2}d\psi$$

we get, after some manipulation,

$$\frac{d}{ds}-\left(s\frac{dF_2}{ds}\right) = \frac{2s}{1-s^2}\int_{0}^{1}\frac{\log(2z)}{(s^2-z^2)^{1/2}}dz \quad \text{(A7)}$$

with $z=\sin\psi$ as new integration variable. The range from s to 1 gives imaginary terms which do not contribute when the real part is taken. The rest involves

$$\int_{0}^{s}\frac{\log(2z)}{(s^2-z^2)^{1/2}}dz = \frac{\pi}{2}\log s$$

on taking $z=s\sin u$ as a new variable.

The integrations with respect to s needed to recover $F_2(s)$ from (A7) are straightforward and eventually we get

$$F(s) = \frac{\pi}{8}[Li_3(1) - Li_3(s^2) + \log s Li_2(s^2)]. \quad \text{(A8)}$$

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Automatic Rieke Diagram Drawing System

Y. KOSUGI AND Y. NAITO

***Abstract*—The development of a system which provides power-constant loci on a Smith chart automatically is presented. The system consists of phase shifters terminated with p-i-n diodes, and a network analyzer.**

Introduction

In the measurement of microwave oscillators, drawing of the Rieke diagram has been time-consuming. The system proposed here is intended to get power-constant loci on a Smith chart automatically. An impedance variable termination, the main part of the system, is also described in detail. Experimental results both at S band and X band are presented.

Impedance Variable Termination

The impedance variable termination is one of the most important components of the microwave circuit. Hitherto, some attempts have been done to improve the characteristics of the variable termination for the sake of easy handling [1]. But it still requires mechanical adjustments that have prevented the automation of some measurements.

The variable termination described in the following section is a useful element whose impedance can be electrically controlled.

When an ideal phase shifter is terminated at one-port with a variable resistance, as shown in Fig. 1, the input impedance at the other port can cover the area inside the Smith chart.

At microwave frequencies, the impedance of a typical p-i-n diode changes according to the bias current as shown in Fig. 2. For the purpose of making an impedance variable termination with a phase shifter, the variable resistance must range from 50 Ω to ∞ or 0 Ω to 50 Ω. (In this case 50 Ω is taken to be the characteristic impedance of the waveguide.)

When a p-i-n diode is the variable resistance, there are two regions that can be used. One of them is the low-resistance region ($0\,\Omega<r<50\,\Omega$) and the other one is high-resistance region ($50\,\Omega<r<\infty$).

For general use, the impedance variable termination is required not to excite any harmonics. In using a p-i-n diode, care should be taken to prevent harmonics.

In the microwave region, usually a p-i-n diode does not function as a rectifier but can be regarded as a resistance. But in the low-frequency region of UHF, there can be observed some harmonic excitation, and the harmonic excitation may be predicted from the nonlinearity of the static characteristic which can be written

$$R = AI^{\alpha} \quad (1)$$

where R is the dc resistance of the diode (in ohms), I is the bias current (in amperes), and for a typical p-i-n diode $A\simeq1.143$, $\alpha\simeq-0.923$. When a small ac current is superposed on the dc bias current I_0, as shown in (2), some harmonic voltage

$$I = I_0 + I_\omega \sin\omega t \quad (2)$$

will be excited between the two terminals of the diode.

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